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Analysis and Design of Pocklingotn's Equation for any Arbitrary Surface for Radiation

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Abstract— Electromagnetic field radiation mechanism for any arbitrary conducting surface is mathematically computed by solving the pocklington's equation, and which is generalize that can be used to any type of antenna in field theory. Tangential vector on any arbitrary surface is defined with the help of surface equation. Using Lorentz gauge condition, scalar potential is defined in terms of vector potential and scattered electric field is calculated on the arbitrary surface. Mathematical representation of E and H filed for parabolic reflector is also derived.

Index Terms— Electromagnetic scattering, Integral equations, Numerical solutions, Pocklington's equation, parabolic reflector.

I. Introduction

THE integral equation method, with a Moment Method numerical solution, is used first to find the self- and driving-point impedances of any antenna, and mutual impedance of wire type of antennas. This method casts the solution for the induced current in the form of an integral where the unknown induced current density is part of the integrand. Numerical techniques, such as the Moment Method, can then be used to solve the current density. In particular two classical integral equations for linear elements, Pocklington's and Hall´en's Integral equations, is used. Hall´en's equation is usually restricted to the use of a delta-gap voltage source model at the feed of a wire antenna. Pocklington's equation, however, is more general and it is adaptable to many types of feed sources (through alteration of its excitation function or excitation matrix), including a magnetic frill.

The straight, thin, center-driven wire is often used as a transmitting antenna. In theoretical studies, the basic unknown is the current along the antenna, which satisfies a one-dimensional, first-kind, Fredholm-like integral equation usually referred to as "Hallén's" equation, or a corresponding integrodifferential equation called "Pocklington's" equation. Pocklington's integro-differential equation is a staple of thin-wire antenna analysis, and appears in most antenna text books [1]-[3], as well as forming the basis of antenna simulation codes such as the Numerical Electromagnetic Code (NEC) [2]. In 1897 Pocklington deduced its equation for straight structures, and in 1965 Mei, used a heuristically procedure to define it for bent wires [4]; for an arbitrary shaped wire, it is possible to deduce the equation using a formal way, starting from Maxwell equations [6] getting:

$$E^I{}_S = -\frac{j}{\varepsilon\omega}\int_{s'} Is(s')\left[k^2 s.s' + \frac{\partial^2}{\partial s \partial s'}\right]\frac{e^{-jk|r-r'|}}{4\pi|r-r'|}ds' \text{ -- (1)}$$

Where $E^I{}_S$ is the tangential incident electric field. Considering the thin-wire approximation and skin effect, is possible to express the electric field as a linear integration over the arch length s □. The general Pocklington equation (1) can be used for any possible thin wire geometry. The wire's geometry is expressed by the dot product $s.s'$, where $s(s)$ are the unit tangential vector for the wire's axis and $s'.s'$ the same for the parallel curve representing the current filament.

$$s(s) = \frac{dx(s)}{ds}i + \frac{dy(s)}{ds}j + \frac{dz(s)}{ds}k$$

$$s'(s') = \frac{dx'(s')}{ds'}i + \frac{dy'(s')}{ds'}j + \frac{dz'(s')}{ds'}k \text{ -----------------}$$

(2)

The geometry is also expressed by the difference between the vectors $|r-r'|$ as:

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$$R = |R| = |r - r'| = \sqrt{[x(s) - x'(s')]^2 + y(s) - y'(s')]^2 + z(s) - z'(s')]^2} \quad \text{--(3)}$$

Defined all former equations, the work is reduced to find the vectors representing the parallel and axis curves for the considered wire and solve it by Method of Moments [7]

II. THEORY AND FORMULATION

Generalization of a pocklington's equation for any arbitrary surface. Let the surface equation be given by $r(u,v)$ where u and v are free parameter. Define the following tangential vector on the surface [8]-[10].

$e_u(u,v) = \frac{\delta r}{\delta u}$ And $e_v(u,v) = \frac{\delta r}{\delta v}$

The surface element is given by:

$$dS(u,v) = e_u \times e_v du dv$$

The surface current density can be written as:

$$J_s(u,v) = J_u(u,v)e_u(u,v) + J_v(u,v)e_v(u,v)$$

It follows that the vector potential produced by this surface current density is given by [11]

$$A(r) = \frac{\mu}{4\pi}\int_S J_s(u,v).\frac{e^{-jkR(u,v)}}{R(u,v)} dS(u,v)$$

Here, $R(u,v) = |r - r(u,v)|$

We calculate the scalar potential from this making use of the Lorentz gauge condition [12]

$$\Phi = \left(\frac{j}{\omega\mu\varepsilon}\right) divA$$

Now

$$divA = \frac{\mu}{4\pi}\int (Js(u,v), \nabla R(u,v)) G(r,u,v) dS(u,v)$$

Where, $\nabla R(u,v) = \nabla |r - r(u,v)| = n(u,v)$

And $G(r,u,v) = \frac{d\frac{e^{-jkR}}{R}}{dR} = -\left(\frac{jk}{R} + \frac{1}{R^2}\right)e^{-jkR}$

It follows that

$$\Phi = \frac{j}{4\pi\omega\varepsilon}\int (J_s, n)(u,v) G(r,u,v) dS(u,v)$$

So that,

$$E = -\nabla\Phi - j\omega A = -\frac{j}{4\pi\omega\varepsilon}\int (J_s, n)\nabla G(r,u,v) dS(u,v) - \frac{j\omega\mu}{4\pi}\int Js(u,v) H(r,u,v) dS(u,v)$$

Where, $H(r,u,v) = \frac{e^{-jkR(u,v)}}{R(u,v)}$

The electric field, we denote by $E_s(r)$ the subscript s standing for "Scattered" [13]. We can write in component form as: $E_{sk} = \sum_{m=1}^{m=3}\int \psi_{km}(r,u,v) J_m(u,v) du dv$

$k = 1,2,3...$

Where $\psi_{km}(r,u,v) = -\frac{j}{4\pi\omega\varepsilon} G_k(r,u,v) n_m(u,v) - \frac{j\omega\mu}{4\pi} H(r,u,v)\delta_{km}$

The tangential components of the scattered electric field on the given surface (along the e_u and e_v direction) are [14]

$$(e_u, E_s) = \sum_{k=1}^{3} x_{k,u} E_{sk}, (e_v, E_s) = \sum_{k=1}^{3} x_{k,v} E_{sk}$$

This can be used to generalize the pocklington's equation.

III. APPLICATION FOR PARABOLIC REFLECTOR

It has been shown by geometrical optics that if a beam of parallel rays is incident upon a reflector whose geometrical shape is a parabola, the radiation will converge (focus) at a spot which is known as the focal point. A parabola is defined as the locus of a point the ratio of whose distance from a point P and from a line is equal to unity. The point 'P' is called the focus. Equation of a parabola in the $x - y$ plane can be expressed as $y^2 = 4ax$. Focus of such a parabola is given by $P = (a,0)$. The symmetrical point on the parabolic surface is known as the vertex. Rays that emerge in a parallel Formation are usually said to be collimated. A paraboloid is a surface obtained by rotating a parabola about the normal to its apex. Equation of parabola can be given as: -

$$x^2 + y^2 = 4az = \rho^2$$

Where, y= distance at y axis, x= distance at x axis, a = distance of focus point from axis

As, $y^2 = 4ax \quad \rho = \sqrt{4az}$

And $\rho = \sqrt{x^2 + y^2}$

So that $x^2 + y^2 - 4az = 0$

Parabolic cylinders have widely been used as high-gain apertures fed by line sources. The analysis of a parabolic cylinder (single curved) reflector is similar, but considerably simpler than that of a paraboloidal (double curved) reflector. The principal characteristics of aperture amplitude, phase, and

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polarization for a parabolic cylinder, as contrasted to those of a paraboloid, are as follows:

1. The amplitude taper, due to variations in distance from the feed to the surface of the parabolic reflector, is proportional to 1/ρ in a cylinder compared to $\frac{1}{r^2}$ in a paraboloid.

2. The focal region, on which incident plane waves converge, is a line source for a cylinder and a point source for a paraboloid.

3. When the fields of the feed are linearly polarized and which are parallel to the axis of the cylinder, no cross-polarized components are produced by the parabolic cylinder. That is not the case for a paraboloid.

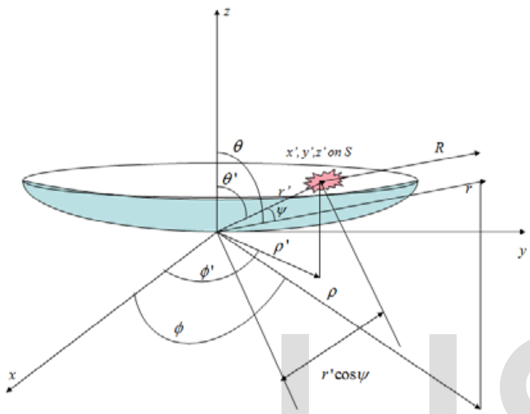

Fig 1. After rotating a parabola about the normal to its apex.

IV. CALCULATION OF INFINITESIMAL AREA

From trigonometry, we have

$$x = \rho\cos\phi$$

$$y = \rho\sin\phi$$ And,

$$z = \frac{\rho^2}{4a}$$

The equation of the paraboloid can be expressed in parametric form as –

$$r(\rho,\phi) = \rho.\cos(\phi)\hat{x} + \rho.\sin(\phi)\hat{y} + (\frac{\rho^2}{4a})\hat{z}$$

Tangent vector on this surface relative to the parametric coordinate (ρ,ϕ) will be

$$e_\rho = \frac{\delta r}{\delta\rho} = \cos(\phi)\hat{x} + \sin(\phi)\hat{y} + (\frac{\rho}{2a})\hat{z}$$

And

$$e_\phi = \frac{1}{\rho}\frac{\delta r}{\delta\phi} = -\sin(\phi)\hat{x} + \cos(\phi)\hat{y}$$

Area element (differential) on the surface is given by –

$$dS(\rho,\phi) = \left|\frac{\delta r}{\delta\rho} x \frac{\delta r}{\delta\phi}\right| d\rho.d\phi$$

Or,

$$dS(\rho,\phi) = \left|e_\rho \times \rho e_\phi\right| d\rho.d\phi$$

Using cross product, i.e. area under the infinitesimal curve of $\left|e_\rho \times \rho e_\phi\right|$,

i.e. $$\begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \cos\phi & \sin\phi & \frac{\rho}{2a} \\ -\rho\sin\phi & \rho\cos\phi & 0 \end{pmatrix}$$

$$e_\rho \times e_\phi = \rho\left|-(\frac{\rho}{2a})\cos(\phi)\hat{x} - (\frac{\rho}{2a})\sin(\phi)\hat{y} + \hat{z}\right|$$

$$\left|e_\rho \times e_\phi\right| = \rho\sqrt{\left(-(\frac{\rho}{2a})\cos(\phi)\right)^2 + \left(-(\frac{\rho}{2a})\sin(\phi)\right)^2 + (1)^2}$$

So that,

$$\left|e_\rho \times e_\phi\right| = \rho\sqrt{1 + \left[\frac{\rho}{2a}\right]^2}$$

Therefore,

$$dS(\rho,\phi) = \rho\sqrt{1 + \rho^2 / 4a^2}\, d\rho d\phi$$

V. CALCULATION OF CURRENT DENSITY

It is well aware that, electric and magnetic representation in terms of fields generated by an electric current source is J and a magnetic current source is M. The procedure requires that the auxiliary electric and magnetic vector potential functions A and F generated, respectively, by J and M are found first. In turn, the corresponding electric and magnetic fields are then determined (E_A, H_A due to A and E_F ,H_F due to F). The total fields are then obtained by the superposition of the individual fields due to A and F (J and M).

As we know that surface current density is given by: $J_S = \hat{a}_x J_x + \hat{a}_y J_y + \hat{a}_z J_z$ Surface current density for paraboloid can be expressed in parametric form as.

$$J(\rho,\phi) = J_\rho e_\rho + J_\phi e_\phi + J_Z e_Z$$

Where ,

$$J_Z e_Z = 0$$

Therefore,

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$$J(\rho,\phi)=$$

$$J_{\rho}(\cos(\phi)\hat{x}+J_{\phi}(-\sin(\phi)\hat{x}+\cos(\phi)\hat{y})+\sin(\phi)\hat{y}+(\frac{\rho}{2a})\hat{z})$$

Combining them in x, y and z coordinates,

$$J(\rho,\phi)=$$

$$[J_{\rho}\cos(\phi)-J_{\phi}\sin(\phi)]\hat{x}+[J_{\rho}\sin(\phi)+J_{\phi}\cos(\phi)]\hat{y}+[\frac{\rho}{2a}J_{\rho}]\hat{z}$$

VI. CALCULATION OF DISTANCE R

Field radiated by a point source varies as, $\frac{e^{-jkr}}{r}$

Where, e^{-jkr} is phase factor and $\frac{1}{r}$ is the amplitude factor

For far field calculation

$R=r-r'\cos\psi$ For phase terms (ψ is the angle between the two vectors r and r' as shown)

$R=r$, For amplitude, and

$$r'\cos\psi=\hat{r}.r'$$

$$r'=\rho'\cos(\phi')\hat{x}+\rho'\sin(\phi')\hat{y}+(\frac{\rho'}{2a})\hat{z}$$

$(r'=\hat{x}x'+\hat{y}y'+\hat{z}z')$

From equation

$$\begin{pmatrix}\hat{x}\\ \hat{y}\\ \hat{z}\end{pmatrix}=\begin{pmatrix}\sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi\\ \sin\theta\sin\phi & \cos\theta\sin\phi & -\sin\theta\\ \cos\theta & -\sin\theta & 0\end{pmatrix}\begin{pmatrix}\hat{r}\\ \hat{\theta}\\ \hat{\phi}\end{pmatrix}$$

$$\hat{r}=\hat{x}\sin\theta\cos\phi+\hat{y}\sin\theta\sin\phi+\hat{z}\cos\theta$$

$$\hat{r}.r'=$$

$$\rho'\sin(\theta)\cos(\phi)\cos(\phi')+\rho'\sin(\theta)\sin(\phi)\sin(\phi')+\frac{\rho'}{2a}\cos\theta$$

$$\hat{r}.r'=r'\cos\psi=\rho'\sin(\theta)\cos(\phi-\phi')+\frac{\rho'}{2a}\cos\theta$$

$$R=r-\hat{r}r'$$

$$R=r-\rho'\sin(\theta)\cos(\phi-\phi')+\frac{\rho'}{2a}\cos\theta$$

This can be written as,

$$R=r-r'\cos\psi$$

Where,

$$r'\cos\psi=\rho'\sin(\theta)\cos(\phi-\phi')+\frac{\rho'}{2a}\cos\theta$$

VII. CALCULATION OF VECTOR POTENTIAL

It is a very common practice in the analysis procedure to introduce auxiliary functions, known as vector potentials, which will aid in the solution of the problems. The most common vector potential function are the A (magnetic vector potential) and F (electric vector potential). To calculate the E and H field, directly from the electric and magnetic current density (J and M) require higher degree of integration [15], which introduce the extra computational time. Therefore, calculation of E and H fields are done through vector potential A and F, through J and M. First integration is done to calculate the vector potential A and F, and then differentiation to obtain E and H fields [16]-[18].

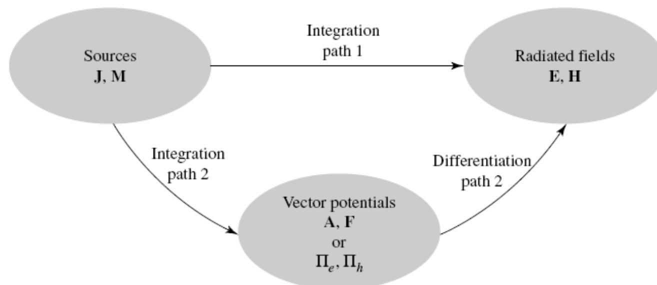

Fig 2. Calculation of E and H field using J and M field.

As per the electromagnetic field theory, we know that magnetic vector potential can be given as:

$$A=\frac{\mu}{4\pi}\iint_{S}J_{s}\frac{e^{-jkR}}{R}ds$$

$$A=\frac{\mu e^{-jkr}}{4\pi}\iint_{S}J_{s}\frac{e^{+jkr'\cos\psi}}{r}ds$$

$$A=\frac{\mu e^{-jkr}}{4\pi}\iint_{S}J_{s}\frac{e^{+jkr'\cos\psi}}{r}\rho\sqrt{1+\rho^{2}/4a^{2}}\,d\rho d\phi$$

$$A=\frac{\mu e^{-jkr}}{4\pi}\iint_{S}\begin{bmatrix}[J_{\rho}\cos(\phi)-J_{\phi}\sin(\phi)]\hat{x}+\\ [J_{\rho}\sin(\phi)+J_{\phi}\cos(\phi)]\hat{y}+\\ [\frac{\rho}{2a}J_{\rho}]\hat{z}\end{bmatrix}$$

Multiply by $\frac{e^{+jkr'\cos\psi}}{r}\rho\sqrt{1+\rho^{2}/4a^{2}}\,d\rho d\phi$

$$A=\frac{\mu e^{-jkr}}{4\pi}\iint_{S}\begin{bmatrix}[J_{\rho}\cos(\phi)-J_{\phi}\sin(\phi)]\hat{x}+\\ [J_{\rho}\sin(\phi)+J_{\phi}\cos(\phi)]\hat{y}+\\ [\frac{\rho}{2a}J_{\rho}]\hat{z}\end{bmatrix}$$

Multiply by

$$\frac{e^{+jk(\rho'\sin(\theta)\cos(\phi-\phi')+\frac{\rho'}{2a}\cos\theta)}}{r}\rho\sqrt{1+\rho^2/4a^2}\,d\rho d\phi$$

With the help of Maxwell equations in homogenous, linear and isotropic medium:

$$\nabla XE = -j\omega\mu H - M$$
$$\nabla XH = j\omega\varepsilon E + J$$
$$\nabla.B = 0$$
$$and$$
$$\nabla.D = 0$$

Where all variables are having their traditional representation. After some mathematical computation (First by putting, $M = 0, J \neq 0$

And then

$J = 0, M \neq 0$

We get.

$$E = -j\omega A - \frac{j}{\omega\mu\varepsilon}\nabla(\nabla.A)$$

And

$$H = -j\omega F - \frac{j}{\omega\mu\varepsilon}\nabla(\nabla.F)$$

VIII. CONCLUSION

The radiation characteristics of any arbitrary surface have been investigated by applying the method of moment and Pocklington's equation, and radiated field equation has been obtained. The integral equation for a parabolic reflector is derived; some properties of integral equation are presented and utilized to reduce the computation of integral equation to some sparse matrix notation. The method is computationally striking, and accurately formulation is demonstrated through illustrative example.

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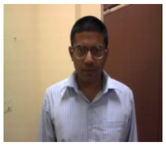

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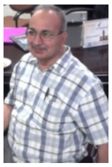

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